



CON JOHNWAY

FERN GOSSOW *PLAYS*
THE HITS OF CONWAY



What goes
next in the
sequence?

THE CROWD WARM-UP

1

11

21

1211

111221

312211

13112221

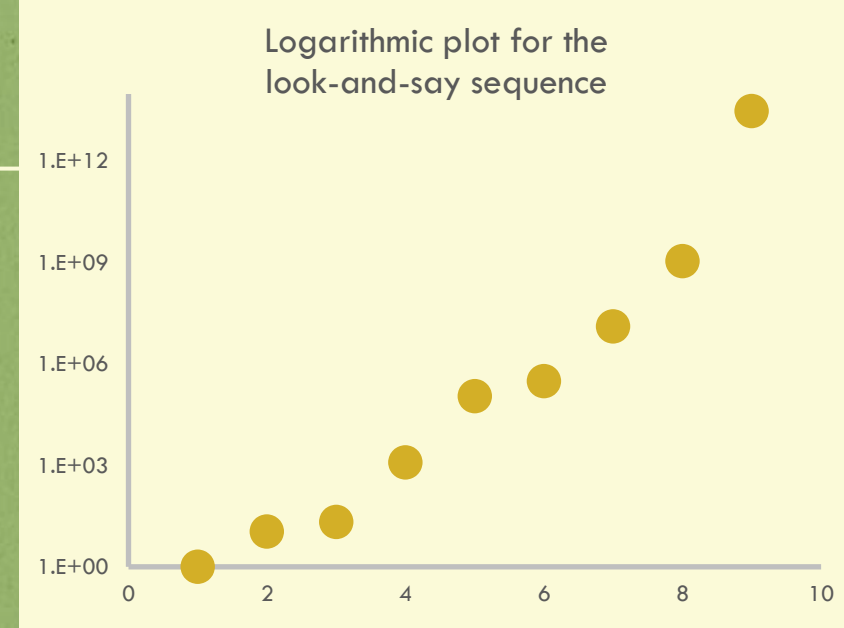
???

The number of digits is approximately exponential, with growth rate

$$\lambda = \lim_{n \rightarrow \infty} \frac{\log_{10}(a_{n+1})}{\log_{10}(a_n)} = 1.3036 \dots$$

(Conway, '86) λ is the unique positive root of the polynomial

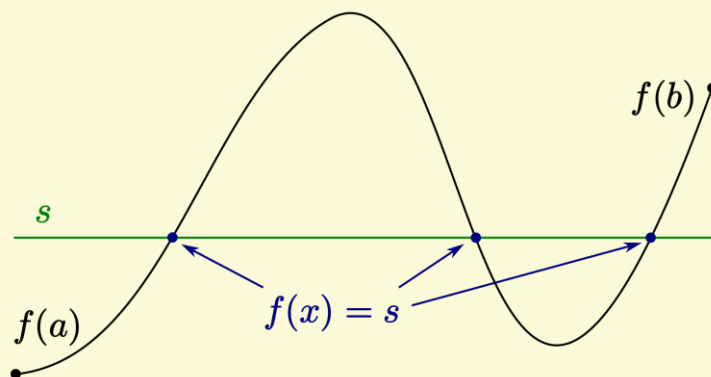
$$\begin{aligned} & x^{71} - x^{69} - 2x^{68} - x^{67} + 2x^{66} + 2x^{65} + x^{64} - x^{63} - x^{62} - x^{61} \\ & - x^{60} + x^{59} + 2x^{58} + 5x^{57} + 3x^{56} - 2x^{55} - 10x^{54} - 3x^{53} - 2x^{52} \\ & + 6x^{51} + 6x^{50} + x^{49} + 9x^{48} - 3x^{47} - 7x^{46} - 8x^{45} - 8x^{44} + 10x^{43} \\ & + 6x^{42} + 8x^{41} - 5x^{40} - 12x^{39} + 7x^{38} - 7x^{37} + 7x^{36} + x^{35} - 3x^{34} \\ & + 10x^{33} + x^{32} - 6x^{31} - 2x^{30} - 10x^{29} - 3x^{28} + 2x^{27} + 9x^{26} \\ & - 3x^{25} + 14x^{24} - 8x^{23} - 7x^{21} + 9x^{20} + 3x^{19} - 4x^{18} - 10x^{17} \\ & - 7x^{16} + 12x^{15} + 7x^{14} + 2x^{13} - 12x^{12} - 4x^{11} - 2x^{10} + 5x^9 + x^7 \\ & - 7x^6 + 7x^5 - 4x^4 + 12x^3 - 6x^2 + 3x - 6 \end{aligned}$$



HOW FAST
DOES THE
SEQUENCE
GROW?

The *intermediate value theorem* says that

if f is a continuous function, then on the interval (a, b) the function takes every value between $f(a)$ and $f(b)$



What about the converse?

if f is a function that takes all values between the endpoints of every interval, is f continuous?



(Conway, '82) *No way*, look at this *wicked* counterexample.

Define $f: \mathbb{R} \rightarrow \mathbb{R}$ as follows:

- For an input x , convert x to base-13.
- Replace the symbols with $\{0,1,2,3,4,5,6,7,8,9, +, -, .\}$
- If from some position onwards, the sequence of digits looks like a well-formed real number, set $f(x)$ to this number.
- If this doesn't work, set it to be zero.

54349589 \rightarrow -34.128 \rightarrow -34.128

54263273 \rightarrow $-31 - +5.$ \rightarrow 5

54556709 \rightarrow $-3.249.$ \rightarrow 0

Division by Three

Peter Doyle, John Conway

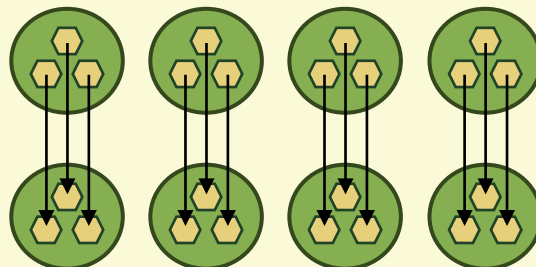
1994

Abstract:

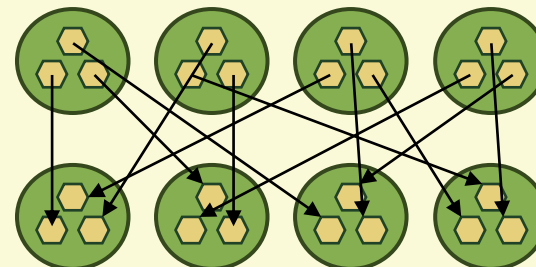
We prove without appeal to the Axiom of Choice that for any sets A and B , if there is a one-to-one correspondence between $3 \times A$ and $3 \times B$ then there is a one-to-one correspondence between A and B . The first such proof, due to Lindenbaum, was announced by Lindenbaum and Tarski in 1926, and subsequently 'lost'; Tarski published an alternative proof in 1949. We argue that the proof presented here follows Lindenbaum's original.

If $\psi: A \rightarrow B$ is a bijection, we can easily construct a bijection

$$\Psi: 3 \times A \rightarrow 3 \times B$$



What about the other way? Our arrows are all mixed up ...



This needs to work for infinite sets, and you can't use cardinality arguments. The proof is tough!

THE CULT CLASSIC

Division by Three

If $\psi: A \rightarrow B$ is a bijection, we can easily construct a bijection

$$\Psi: 3 \times A \rightarrow 3 \times B$$

THE IRONIC ONE
THATS STILL A
RANGER

Peter Doyle,

19

Not that we believe there really are any such things as infinite sets, or that the Zermelo-Fraenkel axioms for set theory are necessarily even *consistent*. Indeed, we're somewhat doubtful whether large natural numbers (like 80^{5000} , or even 2^{200}) exist in any very real sense, and we're secretly hoping that Nelson will succeed in his program for proving that the usual axioms of arithmetic—and hence also of set theory—are inconsistent. (See Nelson [6].) All the more reason, then, for us to stick with methods which, because of their concrete, combinatorial nature, are likely to survive the possible collapse of set theory as we know it today.

Abstract:

We prove without a
Axiom of Choice the
A and B, if there is a
correspondence bet
and $3 \times B$ then there is a one-to-
one correspondence between A
and B. The first such proof, due
to Lindenbaum, was announced
by Lindenbaum and Tarski in
1926, and subse
Tarski published
proof in 1949.
the proof prese
Lindenbaum's original.



This needs to work for infinite sets,

*John Conway collaborated on the research reported here, and has been listed as an author of this work since it was first distributed in 1994. But he has never approved of this exposition, which he regards as full of 'fluff'.

THE ONE PEOPLE THINK HE WROTE

Graduate Texts in Mathematics

John B. Conway

Functions of
One Complex
Variable



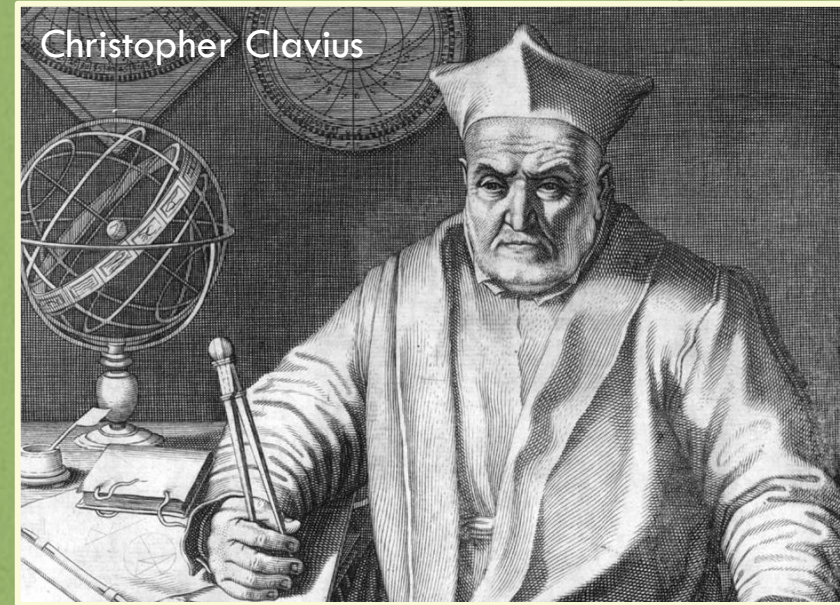
Springer-Verlag
New York Heidelberg Berlin

This very well-written
Springer orange book
on complex analysis.

Observations about the Gregorian calendar

- 1) $365 = 52 \times 7 + 1$
If today is **Thursday**, this date next year will be a **Friday**.
- 2) Every year divisible by 4* has an **extra day** in February.
- 3) Pi day (14th March) in 2000 was a **Tuesday**.

THE ONE YOU DIDN'T REALISE HE WROTE



Dates with the same day

4/4	9/5	Pi day
6/6	5/9	Halloween
8/8	11/7	Boxing day
10/10	7/11	Last of Feb
12/12	Jan 3*	Jan 4*

The Doomsday Algorithm

Let n be the last two digits of the year. Calculate:

$$a = \lfloor n/12 \rfloor \quad b = \text{remainder}$$

$$c = \lfloor b/4 \rfloor \quad d = a + b + c$$

4/4	9/5	Pi day
6/6	5/9	Halloween
8/8	11/7	Boxing day
10/10	7/11	Last of Feb
12/12	Jan 3*	Jan 4*

The **anchor day** for the 21st century is a Tuesday. Every **doomsday** is a

Tuesday + d day

Example

What day will Christmas be next year?

$$2027 \rightarrow 27 = 2 \times 12 + 3$$

$$a = 2, b = 3, c = 0, \text{ so}$$

$$d = 2 + 3 + 0 = 5$$

Since Boxing Day is a doomsday, Christmas is

$$\begin{aligned} & \text{Tuesday} + 5 - 1 \\ & = \text{Tuesday} + 4 \\ & = \text{Saturday} \end{aligned}$$

FRACTRAN

A coding language based on prime decompositions

Multiplication program

3	2	0	0	0	...
---	---	---	---	---	-----

0. $n = 2^3 3^2 5^0 \dots = 72$

1. $n = 72 \cdot \frac{11}{2} = 396$

2. $n = 396 \cdot \frac{455}{33} = 5460$

...

27. $n = 15625 = 5^6$

0	0	6	0	0	...
---	---	---	---	---	-----

State: list of values, encoded as a single positive integer

0	2	1	3	0	...
---	---	---	---	---	-----



$$n = 2^0 3^2 5^1 7^3 11^0 \dots = 15435$$

Program: List of fractions

$$\left(\frac{455}{33}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3} \right)$$

Execution:

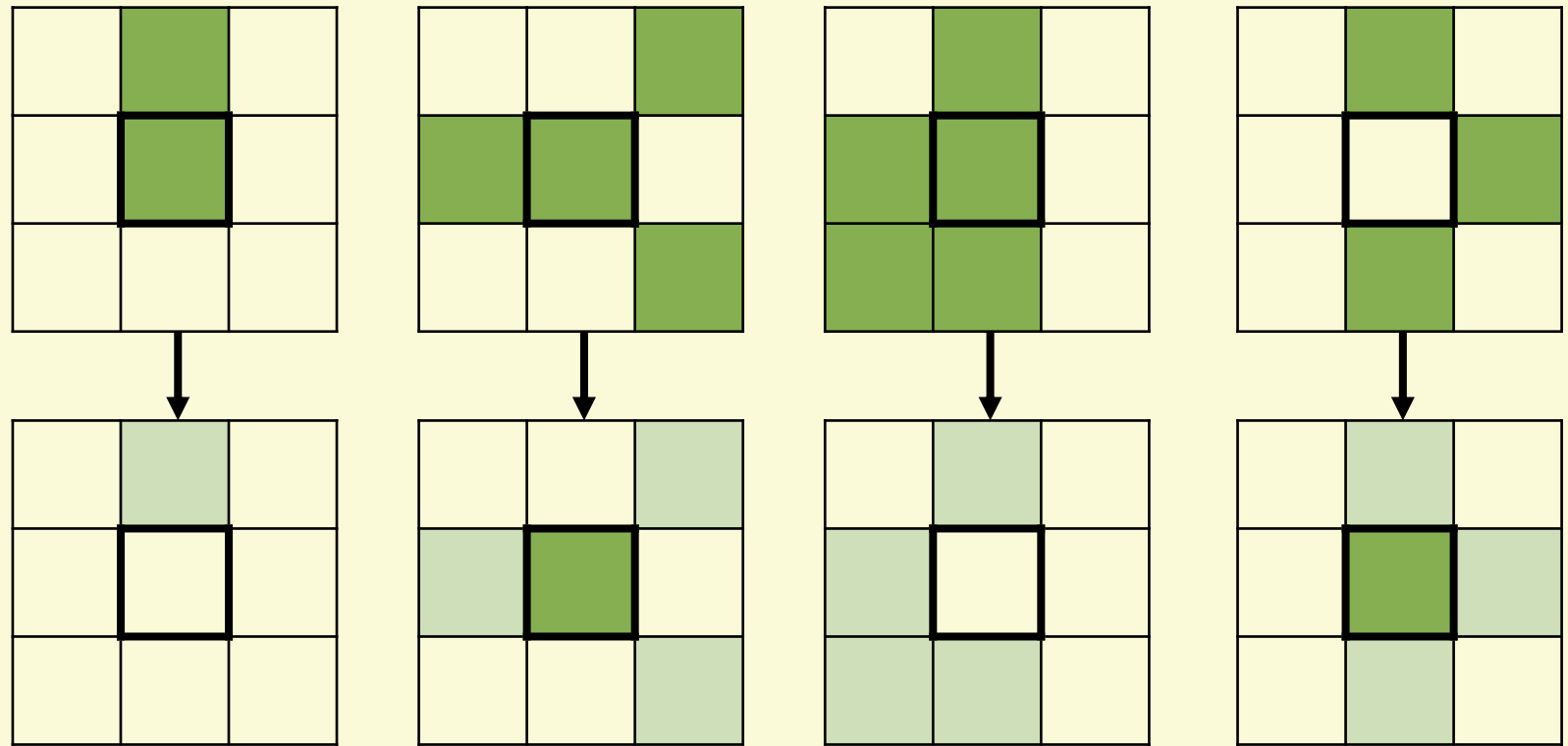
- Find the first fraction f for which nf is an integer
- Replace n with nf
- Repeat until none left

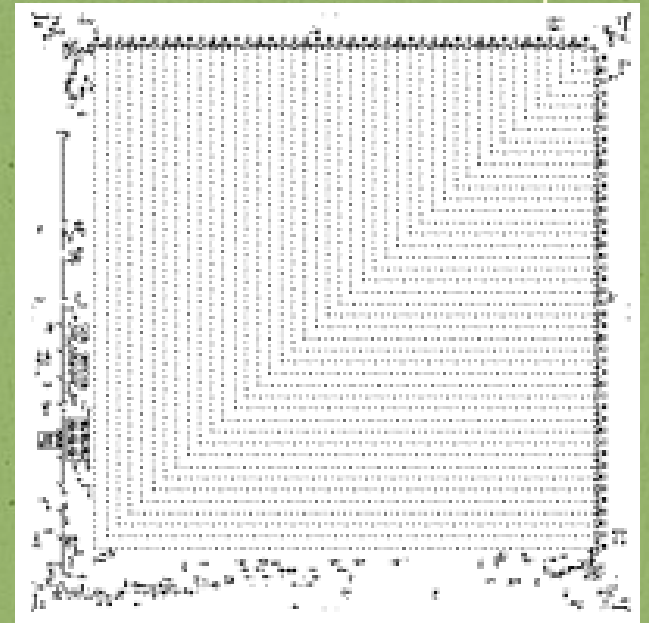
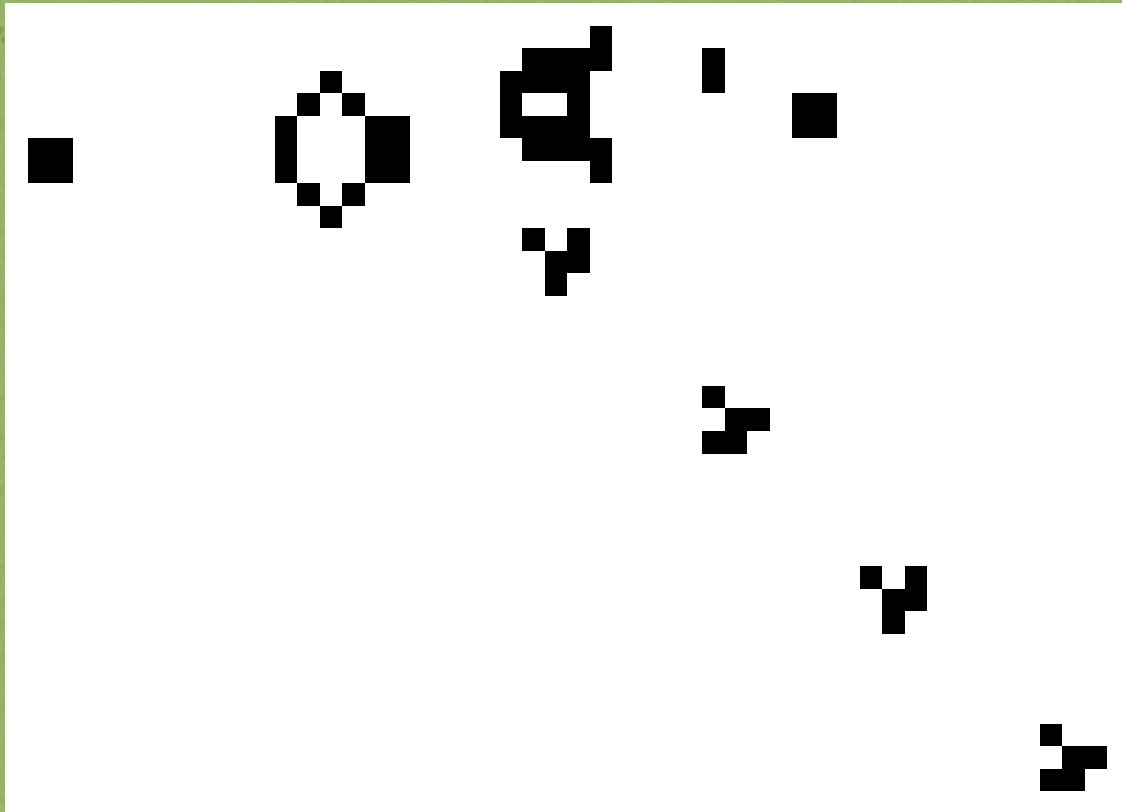
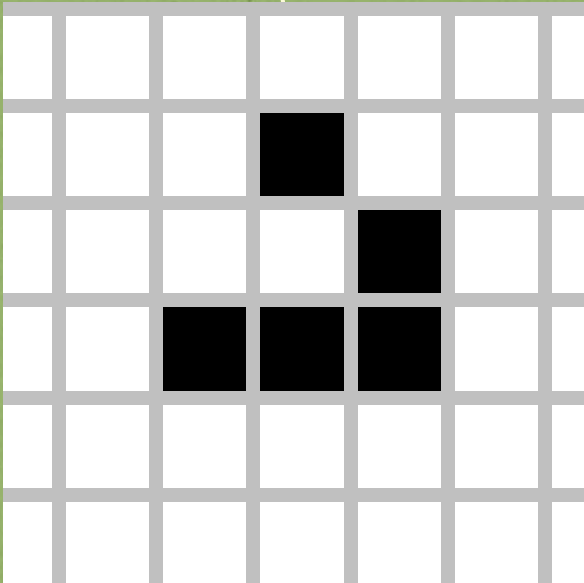
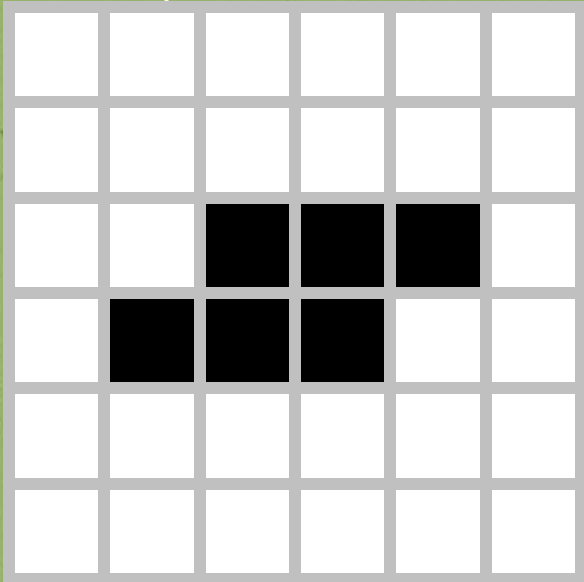
THE ELECTRONIC REMIX

THE ONE HE IS MOST FAMOUS FOR, BUT IS SICK OF

Conway's Game of Life

This is a *zero-player game* played on \mathbb{Z}^2 with the following rules





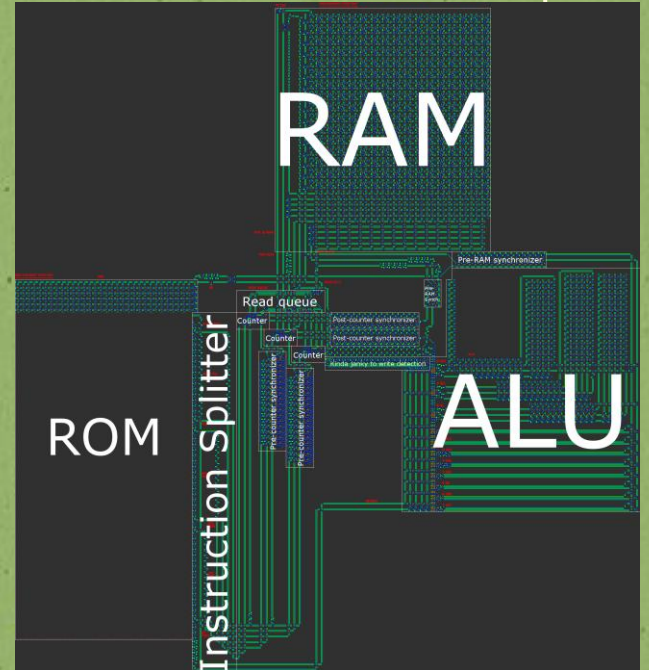
Build a working game of Tetris in Conway's Game of Life

Asked 12 years, 9 months ago Modified 10 months ago Viewed 330k times

▲ Here is a theoretical question - one that doesn't afford an easy answer in any case, not even the trivial one.

1256 ▼ In Conway's Game of Life, there exist constructs such as the [metapixel](#) which allow the Game of Life to simulate any other Game-of-Life rule system as well. In addition, it is known that the Game of Life is Turing-complete.

🔖 Your task is to build a cellular automaton using the rules of Conway's game of life that will allow for the playing of a game of Tetris.

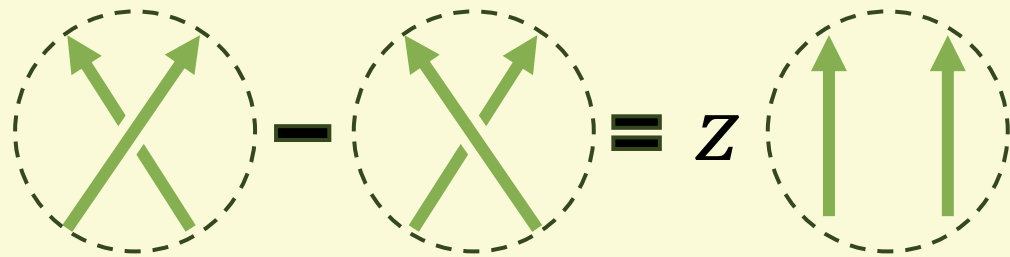


Given two knots, we want to know if they are **equivalent**.

An **invariant** is a function which takes the same value on any equivalent knot.

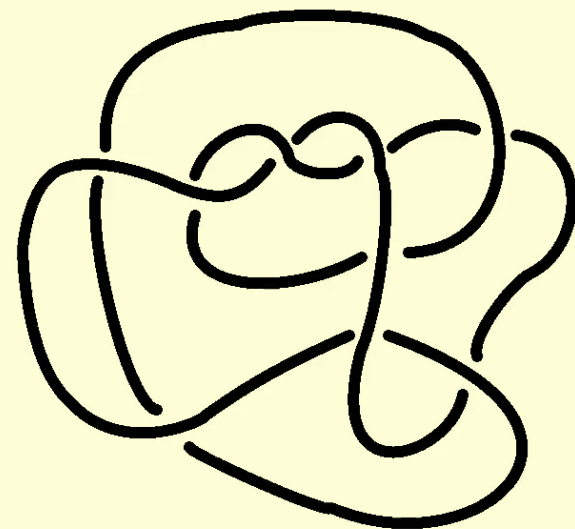
The *Conway polynomial* is a map

$\nabla : \text{knots} \rightarrow \text{polynomials}$



THE REMASTERED DEBUT

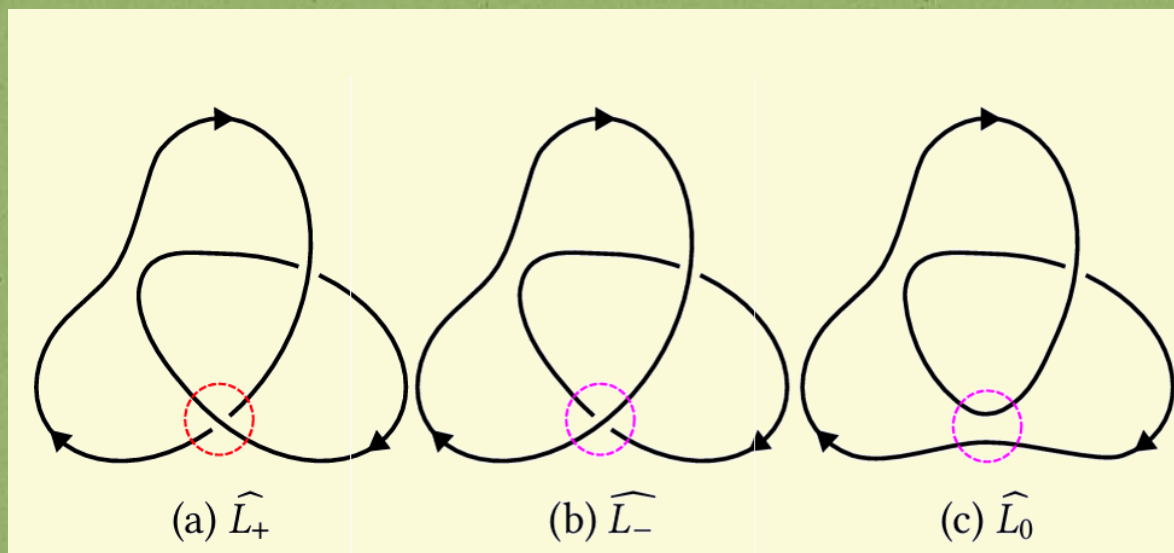
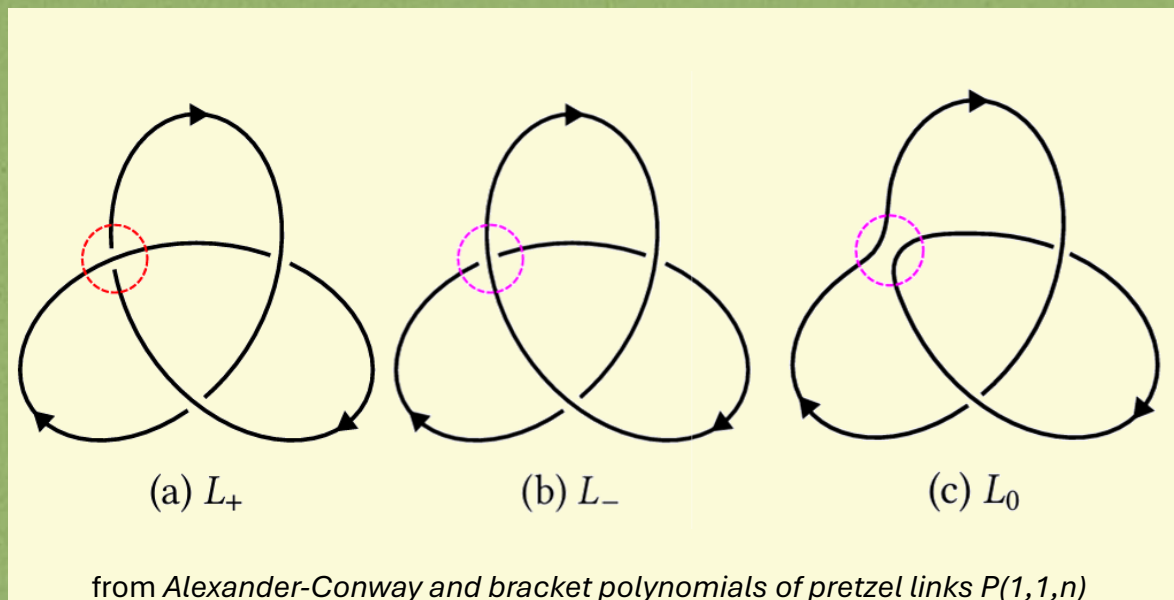
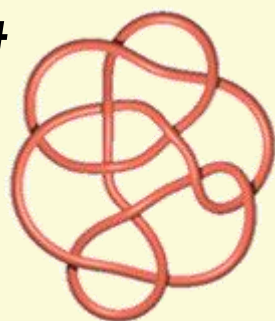
KNOTS AND INVARIANTS



The Alexander-Conway polynomial of the *trefoil* knot is given by

$$\begin{aligned}
 \nabla_{L_+}(z) &= \nabla_{L_-}(z) + z\nabla_{L_0}(z) \\
 &= \nabla_{\bigcirc}(z) + z\nabla_{\widehat{L_+}}(z) \\
 &= \nabla_{\bigcirc}(z) + z[\nabla_{\widehat{L_-}}(z) + z\nabla_{\widehat{L_0}}(z)] \\
 &= \nabla_{\bigcirc}(z) + z[\nabla_{\bigcirc\bigcirc}(z) + z\nabla_{\bigcirc}(z)] \\
 &= 1 + z[0 + z] \\
 \nabla(z) &= 1 + z^2.
 \end{aligned}$$

The *Conway knot* has the same polynomial as the unknot.



After trying to understand endgames of **Go**, Conway invented the following number system:

- 0 is represented $\{ \mid \}$
- If L and R are sets of numbers with $L < R$, then $\alpha = \{L|R\}$ is the 'simplest' number such that $L < \alpha < R$.

THE HIGH CONCEPT ALBUM

$$\begin{array}{ll} \{ 0 \mid \} = 1 & \{ \mid 0 \} = -1 \\ \{ 1 \mid \} = 2 & \{ \mid -1 \} = -2 \\ \{ 2 \mid \} = 3 & \{ \mid -2 \} = -3 \\ & \vdots \end{array}$$

$$\{ 0 \mid 1 \} = \frac{1}{2} \quad \left\{ 0 \mid \frac{1}{2} \right\} = \frac{1}{4}$$

After trying to understand endgames of **Go**, Conway invented the following number system:

- 0 is represented $\{ \mid \}$
- If L and R are sets of numbers with $L < R$, then $\alpha = \{L|R\}$ is the 'simplest' number such that $L < \alpha < R$.

THE HIGH CONCEPT ALBUM

$$\{1,2,3, \dots \mid \} = \omega$$

$$\{\omega \mid \} = \omega + 1$$

$$\{\omega + 1 \mid \} = \omega + 2$$

$$\vdots$$

$$\{\omega, \omega + 1, \omega + 2, \dots \mid \} = 2\omega$$

$$\{\omega, 2\omega, 3\omega, \dots \mid \} = \omega^2$$

$$\{\omega, \omega^2, \omega^3, \dots \mid \} = \omega^\omega$$

$$\{\omega, \omega^\omega, \omega^{\omega^\omega}, \dots \mid \} = \epsilon_0$$

After trying to understand endgames of **Go**, Conway invented the following number system:

- 0 is represented $\{ \mid \}$
- If L and R are sets of numbers with $L < R$, then $\alpha = \{L|R\}$ is the 'simplest' number such that $L < \alpha < R$.

THE HIGH CONCEPT ALBUM

$$\left\{ 0 \mid 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots \right\} = \frac{1}{\omega}$$

$$\{1, 2, 3, \dots \mid \omega\} = \omega - 1$$

$$\{1, 2, 3, \dots \mid \omega, \omega - 1, \dots\} = \frac{\omega}{2}$$

These numbers are a **totally ordered** (proper class) **field**.

We can prove identities like

$$\log \omega = \omega^{1/\omega}$$

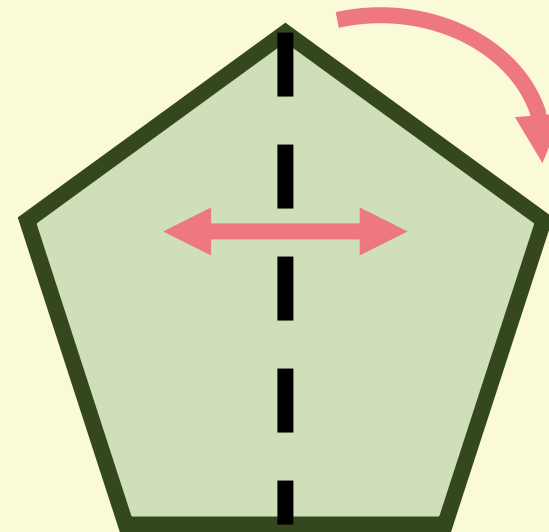
A **group** G describes the symmetries of an object.

- if $g, h \in G$, then $g \circ h \in G$
- $\text{id} \in G$
- if $g \in G$, then $g^{-1} \in G$

A **normal subgroup** is a subgroup $N \leq G$ such that
$$gNg^{-1} = N$$

A group is **simple** if it has no normal subgroups.

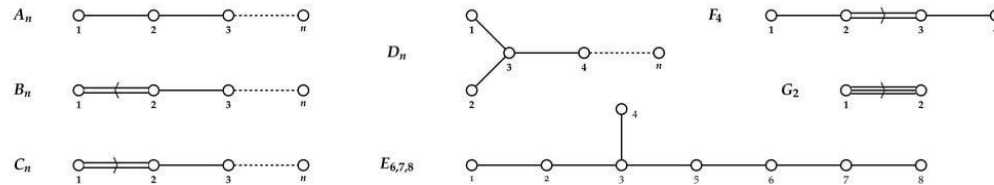
SIMPLE GROUPS AND THE MONSTER



The Periodic Table Of Finite Simple Groups

0, C₁, Z₁
1
1

Dynkin Diagrams of Simple Lie Algebras



$A_1(4), A_1(5)$ A_5 60	$A_2(2)$ $A_1(7)$ 168											${}^2A_3(4)$ $B_2(3)$ 25 920	$C_3(3)$ $C_3(3)$ 4 585 351 680	$D_4(2)$ $D_4(2)$ 174 182 400	${}^2D_4(2^2)$ ${}^2D_4(2^2)$ 197 406 720	$G_2(2)'$ ${}^2A_2(9)$ 6 048	C_2 2
$A_1(9), B_2(2)'$ A_6 360	${}^2G_2(3)'$ $A_1(8)$ 504											$B_2(4)$ $B_2(4)$ 979 200	$C_3(5)$ $C_3(5)$ 228 501 000 000 000	$D_4(3)$ $D_4(3)$ 4 952 179 814 400	${}^2D_4(3^2)$ ${}^2D_4(3^2)$ 10 151 968 619 520	${}^2A_2(16)$ ${}^2A_2(16)$ 62 400	C_3 3
A_7 2 520	$A_1(11)$ 660	$E_6(2)$ 214 841 575 522 005 575 270 400	$E_7(2)$ 7 997 476 042 075 799 739 800 487 262 480 802 918 400	$E_8(2)$ 317 864 751 143 634 806 243 349 190 648 000 300 979 000 482 542 407 451 676 130 989 989 988 800 000	$F_4(2)$ 3 311 126 603 366 400	$G_2(3)$ 4 245 696	${}^3D_4(2^3)$ 211 341 312	${}^2E_6(2^2)$ 76 532 479 683 774 853 939 200	${}^2B_2(2^3)$ 29 120	Tits* ${}^2F_4(2)'$ 17 971 200	${}^2G_2(3^3)$ 10 073 444 472	$B_3(2)$ 1 451 520	$C_4(3)$ $C_4(3)$ 65 784 756 654 489 600	$D_5(2)$ $D_5(2)$ 23 499 295 948 800	${}^2D_5(2^2)$ ${}^2D_5(2^2)$ 25 015 379 558 400	${}^2A_2(25)$ ${}^2A_2(25)$ 126 000	C_7 7
$A_3(2)$ A_8 20 160	$A_1(13)$ 1 092	$E_6(3)$ 7 237 703 347 541 463 210 628 238 395 214 643 200	$E_7(3)$ 1 271 375 236 838 136 742 240 479 751 139 023 644 554 379 203 770 746 254 617 395 200	$E_8(3)$ 349 190 648 000 300 979 000 482 542 407 451 676 130 989 989 988 800 000	$F_4(3)$ 5 734 420 792 816 671 844 761 600	$G_2(4)$ 251 596 800	${}^3D_4(3^3)$ 20 560 831 566 912	${}^2E_6(3^2)$ 14 636 855 916 969 495 633 963 120 680 532 377 600	${}^2B_2(2^5)$ 32 537 600	${}^2F_4(2^3)$ 264 905 352 699 586 176 614 400	${}^2G_2(3^5)$ 49 825 657 439 340 552	$B_2(5)$ 4 680 000	$C_3(7)$ $C_3(7)$ 273 457 218 604 953 600	$D_4(5)$ $D_4(5)$ 8 911 539 000	${}^2D_4(4^2)$ ${}^2D_4(4^2)$ 67 536 471 195 648 000	${}^2A_3(9)$ ${}^2A_3(9)$ 3 265 920	C_{11} 11
A_9 181 440	$A_1(17)$ 2 448	$E_6(4)$ 85 528 710 781 342 640 103 833 619 055 142 765 466 746 800 000	$E_7(4)$ 111 331 438 114 940 385 379 323 475 844 218 044 309 522 133 000 307 231 743 243 304 380 000 000 000	$E_8(4)$ 349 190 648 000 300 979 000 482 542 407 451 676 130 989 989 988 800 000	$F_4(4)$ 19 009 825 523 840 945 451 297 669 120 000	$G_2(5)$ 5 859 000 000	${}^3D_4(4^3)$ 67 802 350 642 790 400	${}^2E_6(4^2)$ 85 496 376 147 617 709 485 896 772 387 584 963 095 360 000 000	${}^2B_2(2^7)$ 34 093 383 680	${}^2F_4(2^5)$ 1 318 633 155 799 391 447 702 161 609 782 732 560 000	${}^2G_2(3^7)$ 239 189 910 264 352 349 332 632	$B_2(7)$ 138 297 600	$C_3(9)$ $C_3(9)$ 54 025 731 402 499 584 000	$D_5(3)$ $D_5(3)$ 1 289 512 799 941 305 139 200	${}^2D_4(5^2)$ ${}^2D_4(5^2)$ 17 880 203 250 000 000 000	${}^2A_2(64)$ ${}^2A_2(64)$ 5 515 776	C_{13} 13
A_n $n!$ 2	$PSL_{n+1}(q), L_{n+1}(q)$ $A_n(q)$	$E_6(q)$ $\frac{q^6(q^6-1)(q^6-1)(q^6-1)}{(q^2-1)(q^2-1)(q^2-1)(q^2-1)}$	$E_7(q)$ $\frac{q^7(q^7-1)(q^7-1)(q^7-1)(q^7-1)}{(2q^2-1) \prod_{i=2}^6 (q^i-1)}$	$E_8(q)$ $\frac{q^8(q^8-1)(q^8-1)(q^8-1)(q^8-1)(q^8-1)(q^8-1)(q^8-1)}{(q^2-1)(q^2-1)(q^2-1)(q^2-1)(q^2-1)(q^2-1)(q^2-1)(q^2-1)}$	$F_4(q)$ $\frac{q^4(q^4-1)(q^4-1)(q^4-1)(q^4-1)}{(q^2-1)(q^2-1)(q^2-1)(q^2-1)}$	$G_2(q)$ $q^6(q^6-1)(q^2-1)$	${}^3D_4(q^3)$ $\frac{q^{12}(q^6+q^4-1)}{(q^2-1)(q^2-1)}$	${}^2E_6(q^2)$ $\frac{q^{12}(q^6-1)(q^6+1)(q^6-1)}{(q^2-1)(q^2+1)(q^2-1)}$	${}^2B_2(2^{2n+1})$ $q^2(q^2+1)(q-1)$	${}^2F_4(2^{2n+1})$ $\frac{q^{12}(q^6+1)(q^6-1)}{(q^2+1)(q^2-1)}$	${}^2G_2(3^{2n+1})$ $q^4(q^4+1)(q-1)$	$O_{2n+1}(q), O_{2n+1}(q)$ $B_n(q)$ $\frac{q^n}{(2q-1)} \prod_{i=1}^n (q^i-1)$	$PSp_{2n}(q)$ $C_n(q)$ $\frac{q^n}{(2q-1)} \prod_{i=1}^n (q^i-1)$	$O_{2n}^+(q)$ $D_n(q)$ $\frac{q^{n(n-1)}(q-1)}{(q^2-1)} \prod_{i=1}^{n-1} (q^i-1)$	$O_{2n}^-(q)$ ${}^2D_n(q^2)$ $\frac{q^{n(n-1)}(q^2-1)}{(q^2+1)} \prod_{i=1}^{n-1} (q^i-1)$	$PSU_{n+1}(q)$ ${}^2A_n(q^2)$ $\frac{q^{n(n+1)} \prod_{i=1}^n (q^i+1)}{(q^2+1) \prod_{i=1}^n (q^i-1)}$	Z_p C_p p

- Alternating Groups
- Classical Chevalley Groups
- Chevalley Groups
- Classical Steinberg Groups
- Steinberg Groups
- Suzuki Groups
- Ree Groups and Tits Group*
- Sporadic Groups
- Cyclic Groups

Alternates†
Symbol
Order‡

M_{11} 7 920	M_{12} 95 040	M_{22} 443 520	M_{23} 10 200 960	M_{24} 244 823 040	$J(1), J(11)$ J_1 175 560	HJ J_2 604 800	HJM J_3 50 232 960	J_4 86 775 571 046 077 562 880	HS 44 352 000	McL 898 128 000	He 4 030 387 200	Ru 145 926 144 000
-------------------	--------------------	---------------------	------------------------	-------------------------	-----------------------------------	--------------------------	------------------------------	--	--------------------	----------------------	-----------------------	-------------------------

*The Tits group ${}^2F_4(2)'$ is not a group of Lie type, but is the (index 2) commutator subgroup of ${}^2F_4(2)$. It is usually given honorary Lie type status.

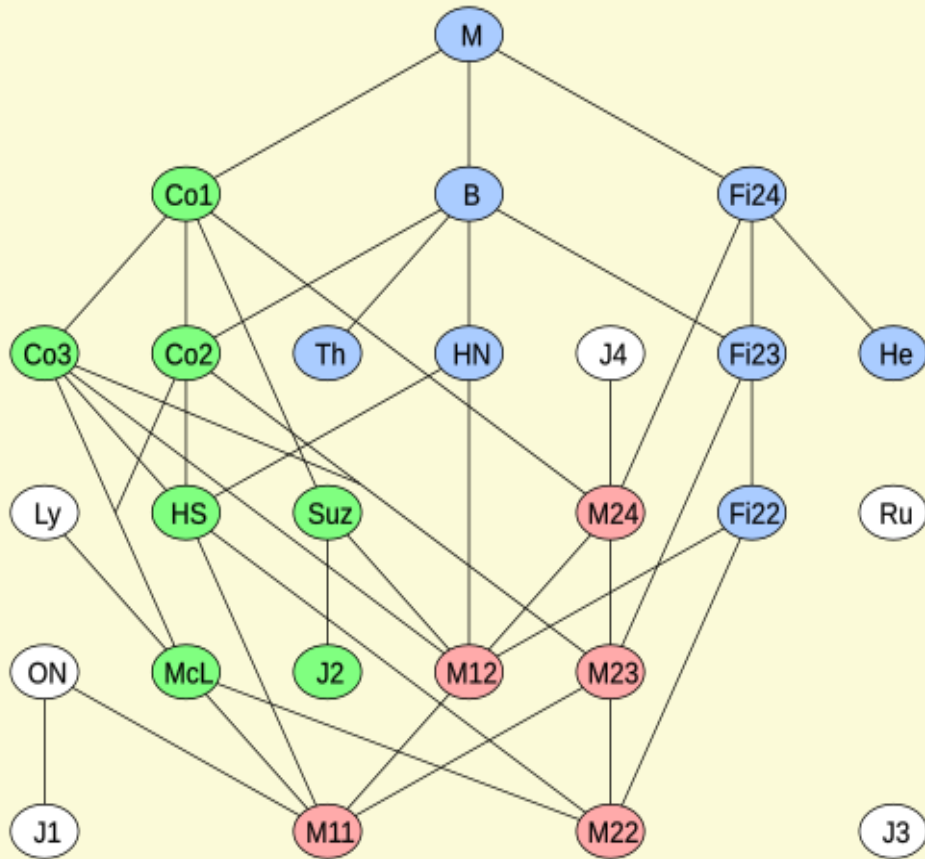
†For sporadic groups and families, alternate names in the upper left are other names by which they may be known. For specific non-sporadic groups these are used to indicate isomorphisms. All such isomorphisms appear on the table except the family $B_n(2^n) \cong C_n(2^{2n})$.

The groups starting on the second row are the classical groups. The sporadic Suzuki group is unrelated to the families of Suzuki groups.

‡Finite simple groups are determined by their order with the following exceptions:
 $B_n(q)$ and $C_n(q)$ for q odd, $n > 2$;
 $A_6 \cong A_3(2)$ and $A_3(4)$ of order 20160.

Sz 448 345 497 600	$O'NS, O-S$ $O'N$ 460 815 505 920	3 Co_3 495 766 656 000	2 Co_2 42 305 421 312 000	1 Co_1 4 157 776 806 543 360 000	F_3, D HN 273 030 912 000 000	LyS Ly 51 765 179 004 000 000	F_3, E Th 90 745 943 887 872 000	$M(22)$ Fi_{22} 64 561 751 654 400	$M(23)$ Fi_{23} 4 089 470 473 293 004 800	$F_3, M(24)'$ Fi'_{24} 1 255 205 709 190 661 721 292 800	F_2 B 4 154 781 403 226 426 191 177 360 344 000 000	F_4, M_1 M 808 017 424 794 512 875 886 489 904 961 710 757 007 754 546 000 000 000
-------------------------	---	-------------------------------------	--	--	--	--	---	--	--	---	--	--

$|M| = 8080174247945128758864599049617$
 10757005754368000000000



A **matrix representation** of a group is a map $g \mapsto [g]$, an $n \times n$ matrix over \mathbb{C} with

$$g \circ h = [g][h].$$

For the Monster Group, the smallest rep has $n = 196883$.

In the theory of modular forms, the **j -invariant** is a modular form with Fourier expansion

$$j(\tau) = q^{-1} + 196884q + 21493760q^2 + \dots$$

Conway did early work on the theory connecting these values, completed by Borcherds ('92).

Other things I didn't mention

- Geometry
Conway polyhedral notation, the grand antiprism, tesellations
- Combinatorial game theory
Sprouts, Philosopher's Football, Domineering strategy
- Number systems
Chained arrow notation, icosians, Waring's conjecture for $(5,37)$
- Angel vs Devil problem
- Chained arrow notation
- Thrackle conjecture
- Free will theorem



THANKS
FOR
LISTENING