CON JOHNWAY

FERN GOSSOW *PLAYS THE HITS* OF CONWAY

What goes next in the sequence?

THE CROWD WARM-UP

1

11

21

1211

111221

312211

13112221

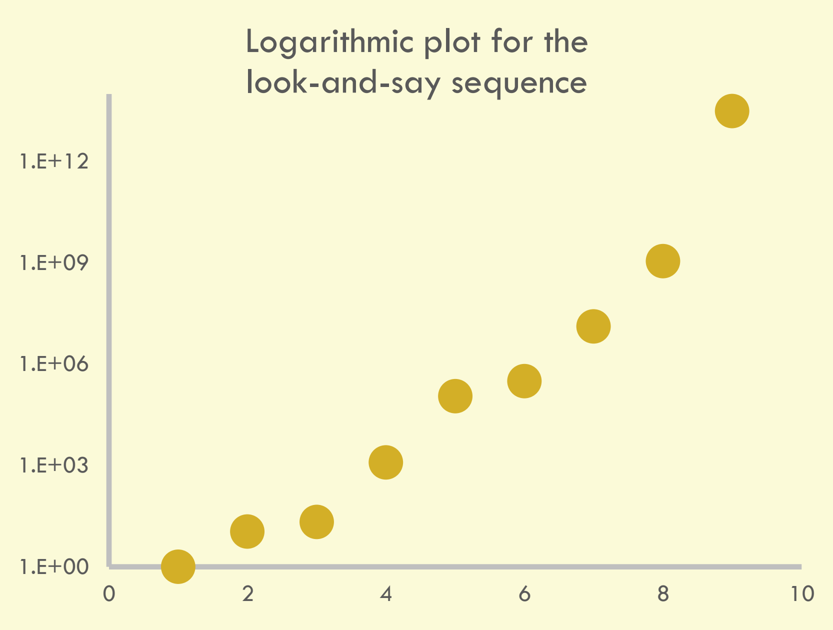
???

The number of digits is approximately exponential, with growth rate

$$\lambda = \lim_{n\to\infty} \frac{\log_{10}(a_{n+1})}{\log_{10}(a_n)} = 1.3036\ldots$$

(Conway, '86) λ is the unique positive root of the polynomial

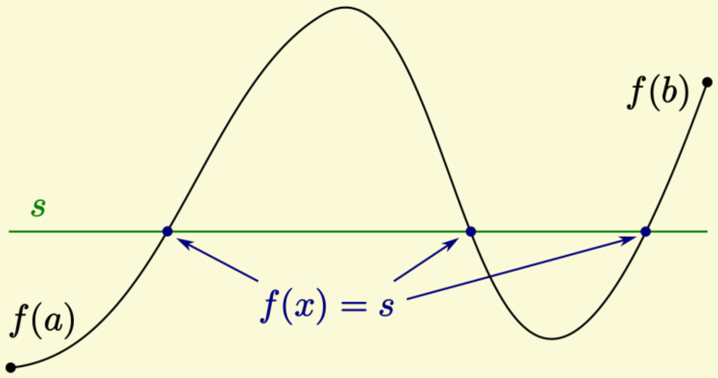
$$x^{71} - x^{69} - 2x^{68} - x^{67} + 2x^{66} + 2x^{65} + x^{64} - x^{63} - x^{62} - x^{61} - x^{60} + x^{59} + 2x^{58} + 5x^{57} + 3x^{56} - 2x^{55} - 10x^{54} - 3x^{53} - 2x^{52} + 6x^{51} + 6x^{50} + x^{49} + 9x^{48} - 3x^{47} - 7x^{46} - 8x^{45} - 8x^{44} + 10x^{43} + 6x^{42} + 8x^{41} - 5x^{40} - 12x^{39} + 7x^{38} - 7x^{37} + 7x^{36} + x^{35} - 3x^{34} + 10x^{33} + x^{32} - 6x^{31} - 2x^{30} - 10x^{29} - 3x^{28} + 2x^{27} + 9x^{26} - 3x^{25} + 14x^{24} - 8x^{23} - 7x^{21} + 9x^{20} + 3x^{19} - 4x^{18} - 10x^{17} - 7x^{16} + 12x^{15} + 7x^{14} + 2x^{13} - 12x^{12} - 4x^{11} - 2x^{10} + 5x^{9} + x^{7} - 7x^{6} + 7x^{5} - 4x^{4} + 12x^{3} - 6x^{2} + 3x - 6$$



HOW FAST DOES THE SEQUENCE GROW?

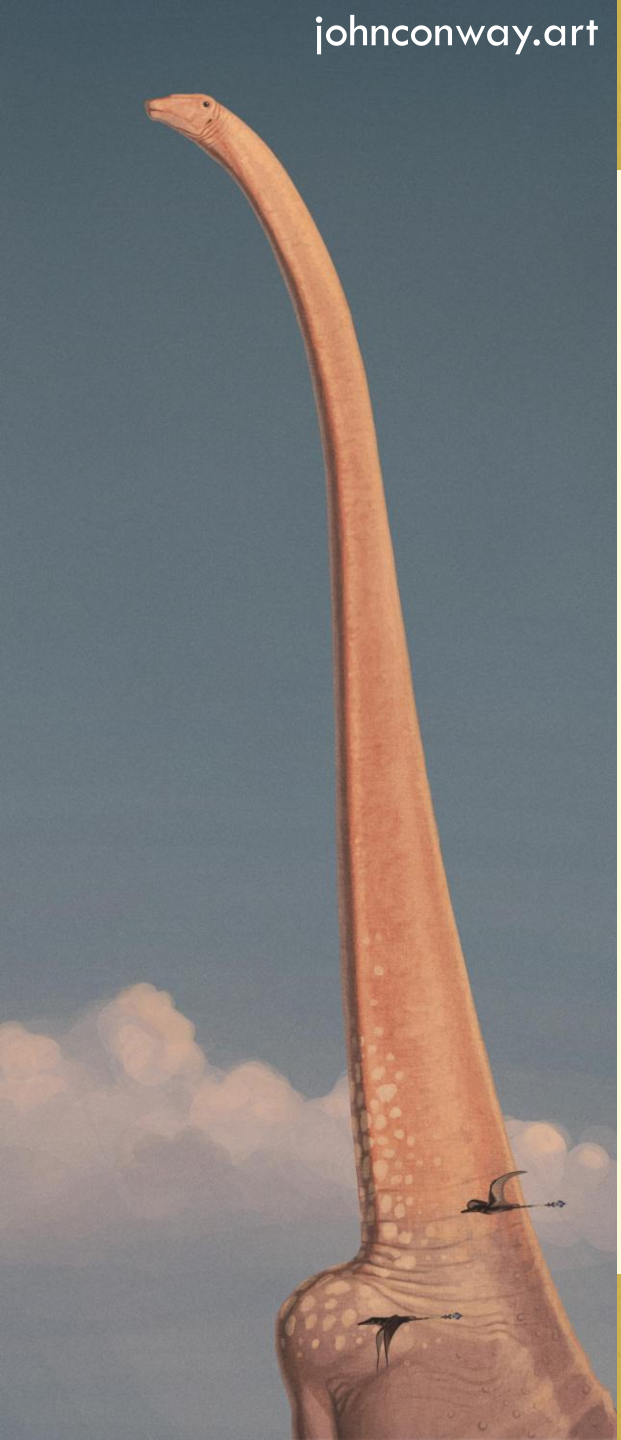
The *intermediate value theorem* says that

> if f is a continuous function, then on the interval (a, b) the function takes every value between $f(a)$ and $f(b)$



What about the converse?

> if f is a function that takes all values between the endpoints of *every* interval, is f continuous?



(Conway, '82) *No way*, look at this *wicked* counterexample.

Define $f : \mathbb{R} \to \mathbb{R}$ as follows:

- For an input x, convert x to base-13.
- Replace the symbols with $\{0,1,2,3,4,5,6,7,8,9,+,-,\ .\}$
- If from some position onwards, the sequence of digits looks like a well-formed real number, set $f(x)$ to this number.
- If this doesn't work, set it to be zero.

54349589	$\to$	-34.128	$\to$	-34.128
54263273	$\to$	$-31-+5.$	$\to$	5
54556709	$\to$	$-3.249.$	$\to$	0

Division by Three

Peter Doyle, John Conway

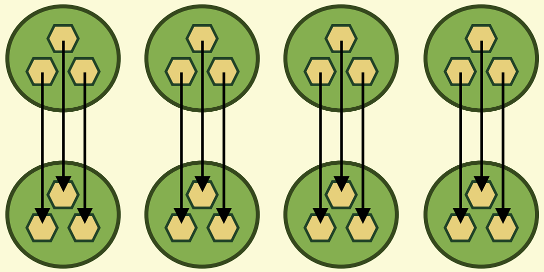
1994

Abstract:

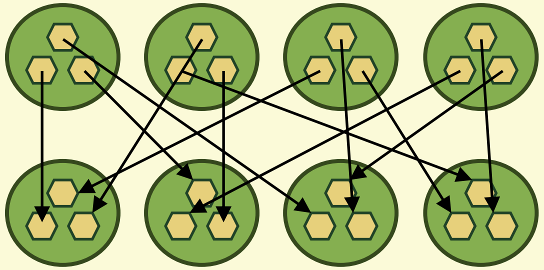
We prove without appeal to the Axiom of Choice that for any sets A and B, if there is a one-to-one correspondence between 3 × A and 3 × B then there is a one-to-one correspondence between A and B. The first such proof, due to Lindenbaum, was announced by Lindenbaum and Tarski in 1926, and subsequently 'lost'; Tarski published an alternative proof in 1949. We argue that the proof presented here follows Lindenbaum's original.

If $\psi: A \to B$ is a bijection, we can easily construct a bijection

$$\Psi: 3 \times A \to 3 \times B$$



What about the other way? Our arrows are all mixed up …



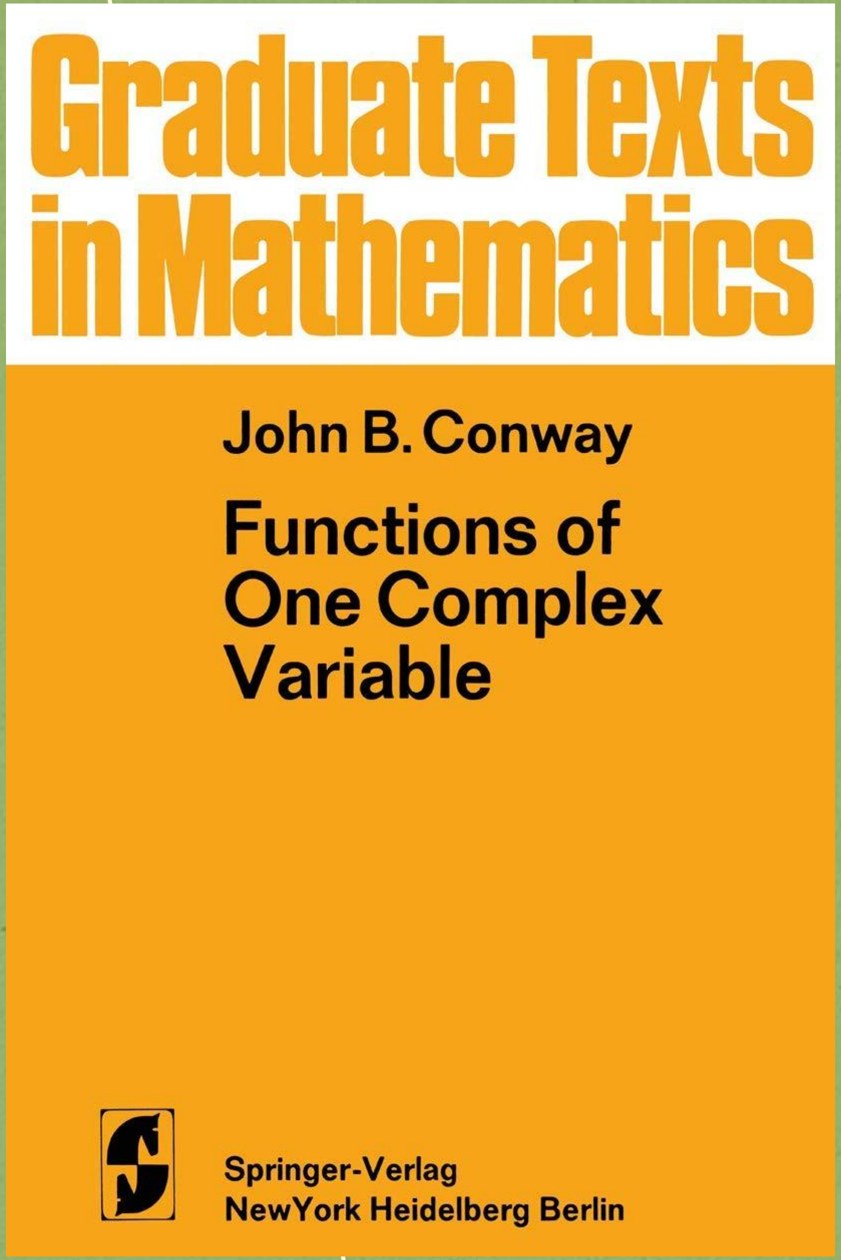
This needs to work for infinite sets, and you can't use cardinality arguments. The proof is tough!

THE CULT CLASSIC

Not that we believe there really are any such things as infinite sets, or that the Zermelo-Fraenkel axioms for set theory are necessarily even *consistent*. Indeed, we're somewhat doubtful whether large natural numbers (like 80^{5000}, or even 2^{200}) exist in any very real sense, and we're secretly hoping that Nelson will succeed in his program for proving that the usual axioms of arithmetic—and hence also of set theory—are inconsistent. (See Nelson [6].) All the more reason, then, for us to stick with methods which, because of their concrete, combinatorial nature, are likely to survive the possible collapse of set theory as we know it today.

*John Conway collaborated on the research reported here, and has been listed as an author of this work since it was first distributed in 1994. But he has never approved of this exposition, which he regards as full of 'fluff'.

THE ONE PEOPLE THINK HE WROTE



This very well-written Springer orange book on complex analysis.

Observations about the Gregorian calendar

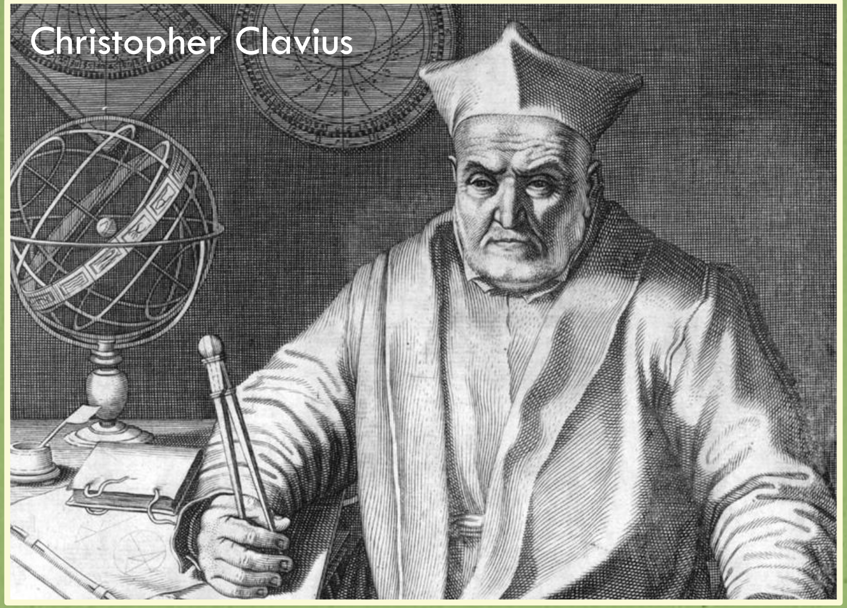
- 1) 365 = 52 x 7 + 1

  If today is **Thursday,** this date next year will be a **Friday.**

- 2) Every year divisible by 4* has an **extra day** in February.

- 3) Pi day (14th March) in 2000 was a **Tuesday.**

THE ONE YOU DIDN`T REALISE HE WROTE



Christopher Clavius

Dates with the same day

4/4	9/5	Pi day
6/6	5/9	Halloween
8/8	11/7	Boxing day
10/10	7/11	Last of Feb
12/12	Jan 3*	Jan 4*

The Doomsday Algorithm

Let n be the last two digits of the year. Calculate:

$a = \lfloor n/12 \rfloor$ $\quad b = \text{remainder}$

$c = \lfloor b/4 \rfloor$ $\quad d = a + b + c$

4/4	9/5	Pi day
6/6	5/9	Halloween
8/8	11/7	Boxing day
10/10	7/11	Last of Feb
12/12	Jan 3*	Jan 4*

The **anchor day** for the 21st century is a Tuesday. Every **doomsday** is a

Tuesday + d day

Example

What day will Christmas be next year?

2027 → 27 = 2 x 12 + 3

$a = 2, b = 3, c = 0,$ so

$d = 2 + 3 + 0 = 5$

Since Boxing Day is a doomsday, Christmas is

$$\text{Tuesday} + 5 - 1 = \text{Tuesday} + 4 = \text{Saturday}$$

FRACTRAN

A coding language based on prime decompositions

State: list of values, encoded as a single positive integer

0	2	1	3	0	…

$$n = 2^0 3^2 5^1 7^3 11^0 \cdots = 15435$$

Program: List of fractions

$$\left(\frac{455}{33}, \frac{11}{13}, \frac{1}{11}, \frac{3}{7}, \frac{11}{2}, \frac{1}{3}\right)$$

Execution:

- Find the first fraction f for which nf is an integer
- Replace n with nf
- Repeat until none left

Multiplication program

3	2	0	0	0	…

0. $n = 2^3 3^2 5^0 \cdots = 72$

1. $n = 72 \cdot \frac{11}{2} = 396$

2. $n = 396 \cdot \frac{455}{33} = 5460$

…

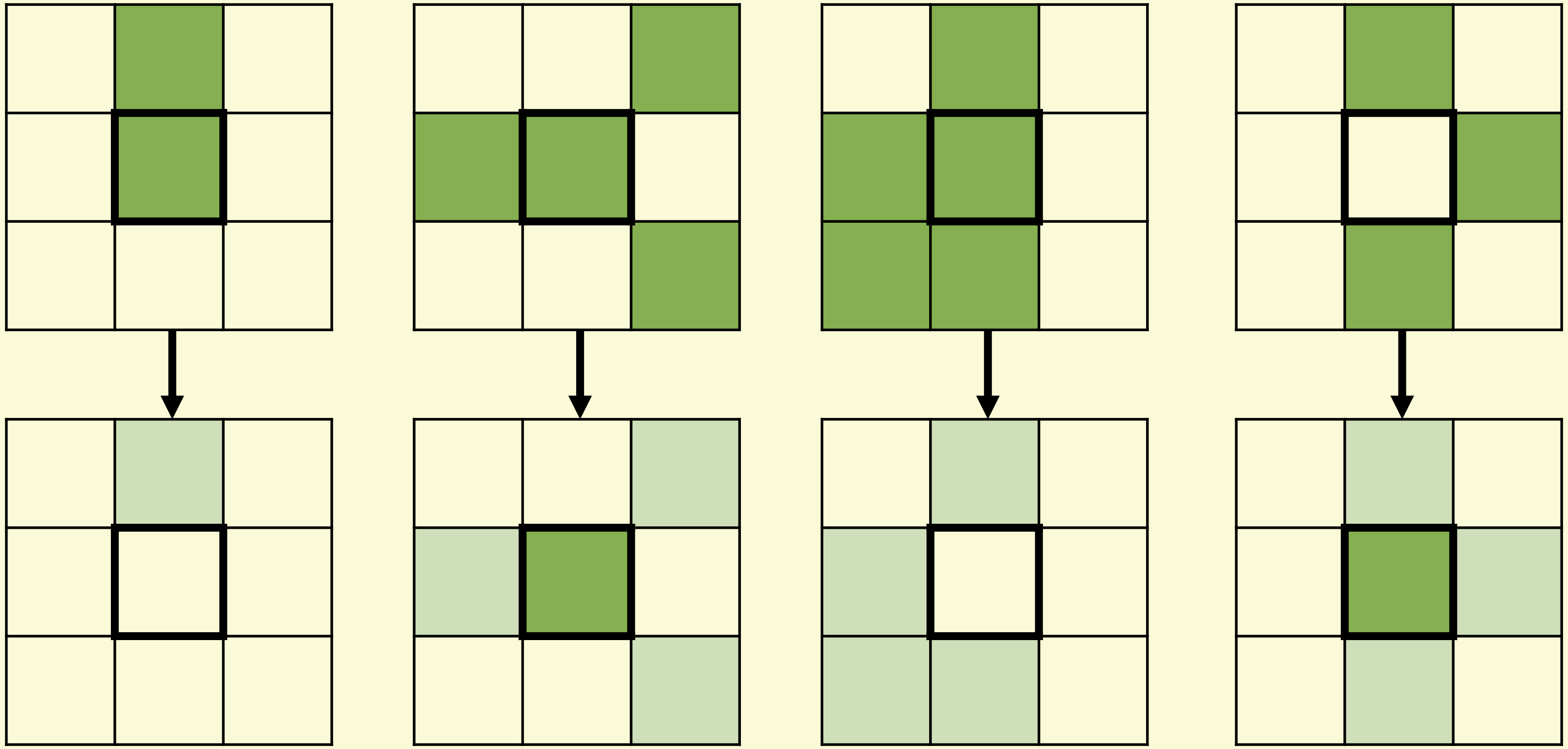
27. $n = 15625 = 5^6$

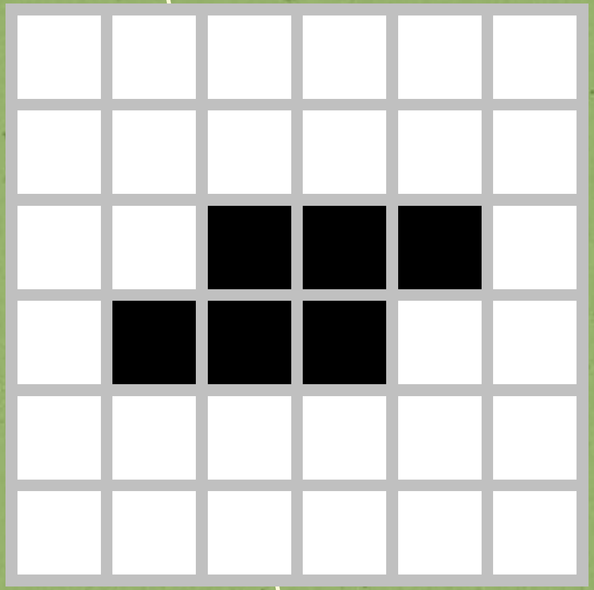
0	0	6	0	0	…

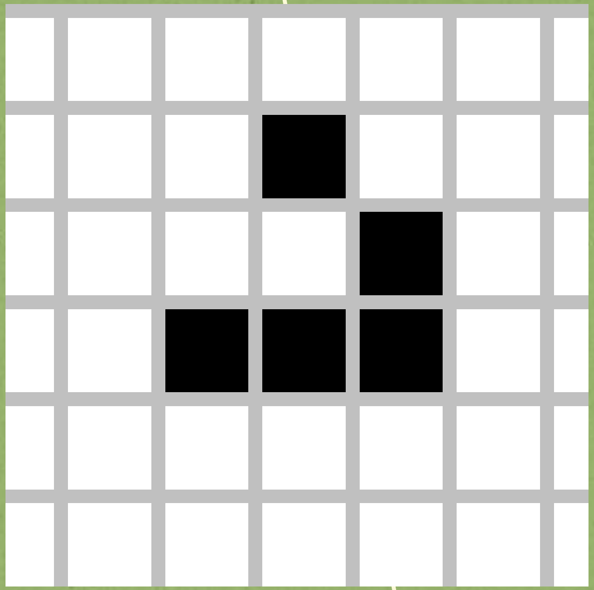
THE ONE HE IS MOST FAMOUS FOR, BUT IS SICK OF

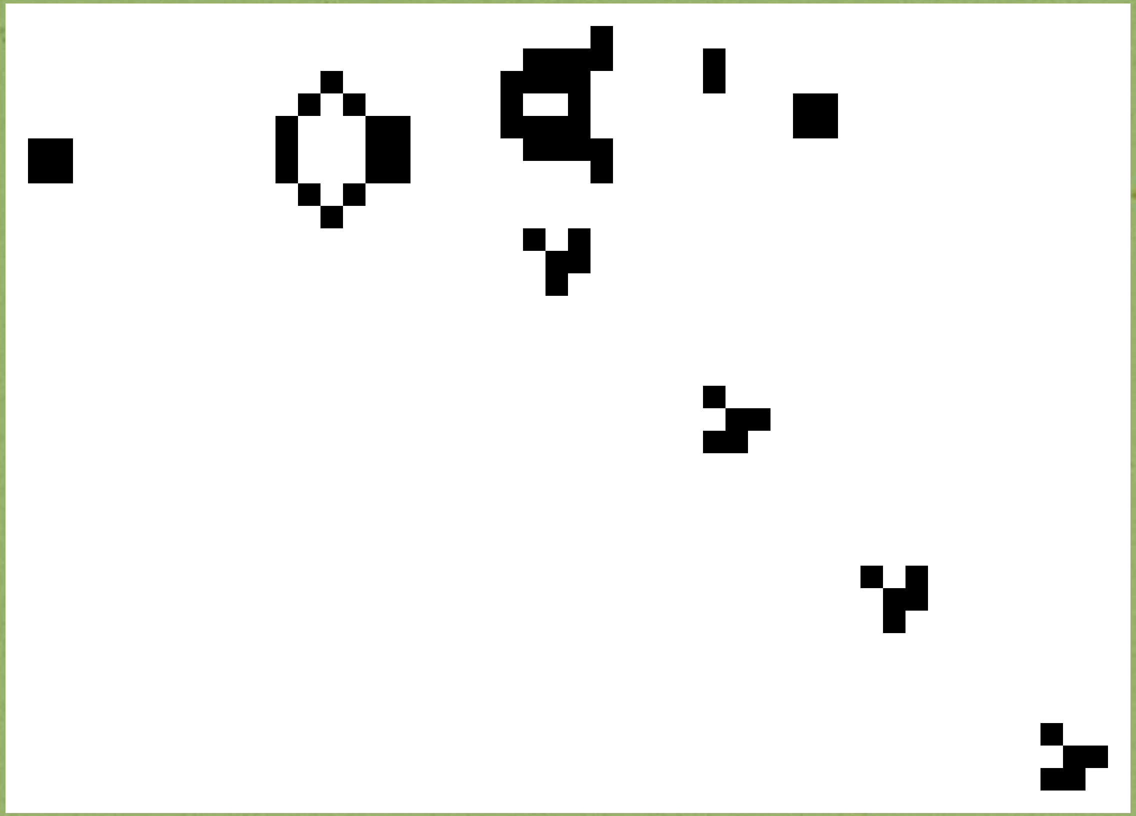
Conway's Game of Life

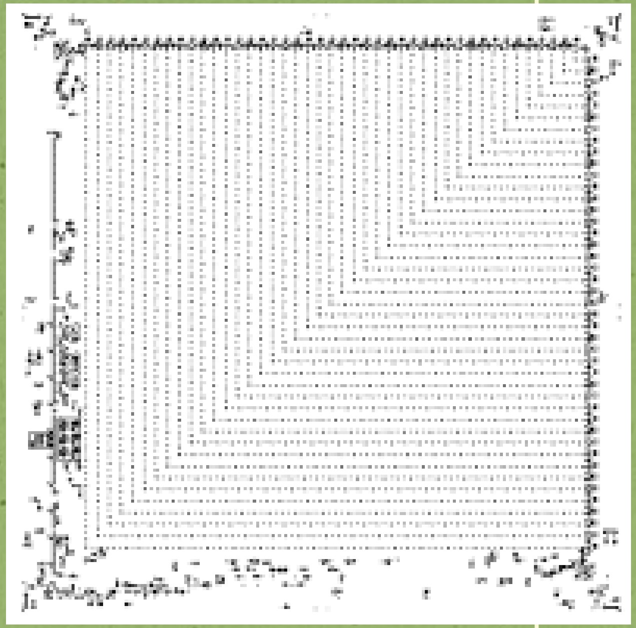
This is a *zero-player game* played on $\mathbb{Z}^2$ with the following rules

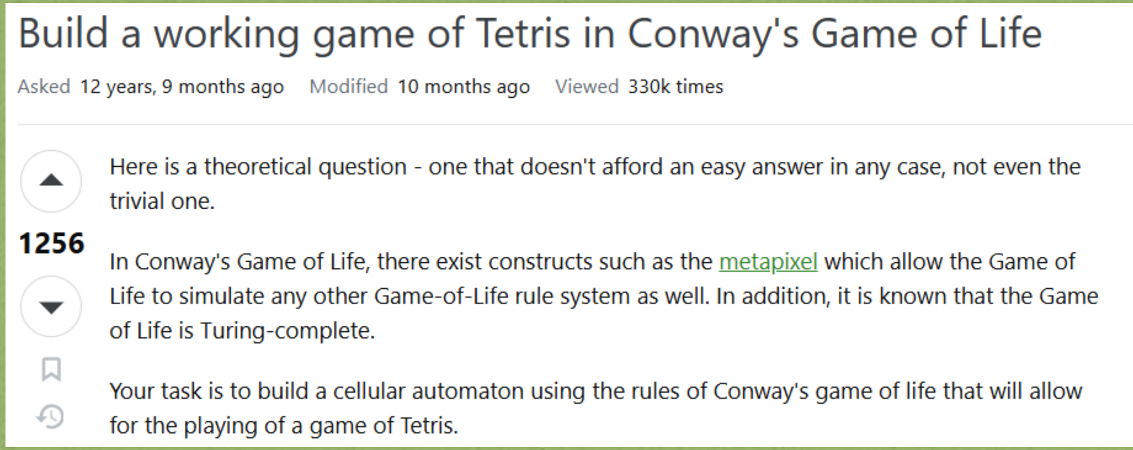












Build a working game of Tetris in Conway's Game of Life

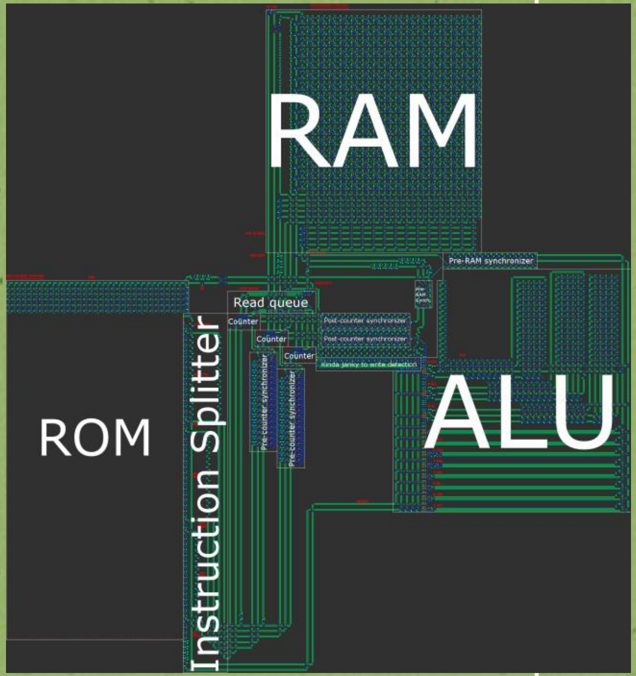
Asked 12 years, 9 months ago Modified 10 months ago Viewed 330k times

1256

Here is a theoretical question - one that doesn't afford an easy answer in any case, not even the trivial one.

In Conway's Game of Life, there exist constructs such as the metapixel which allow the Game of Life to simulate any other Game-of-Life rule system as well. In addition, it is known that the Game of Life is Turing-complete.

Your task is to build a cellular automaton using the rules of Conway's game of life that will allow for the playing of a game of Tetris.



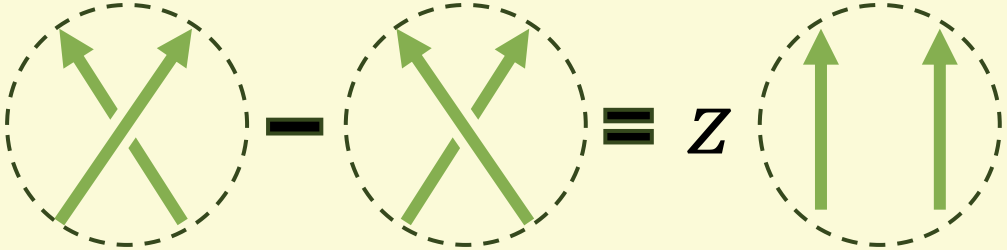
KNOTS AND INVARIANTS

Given two knots, we want to know if they are **equivalent.**

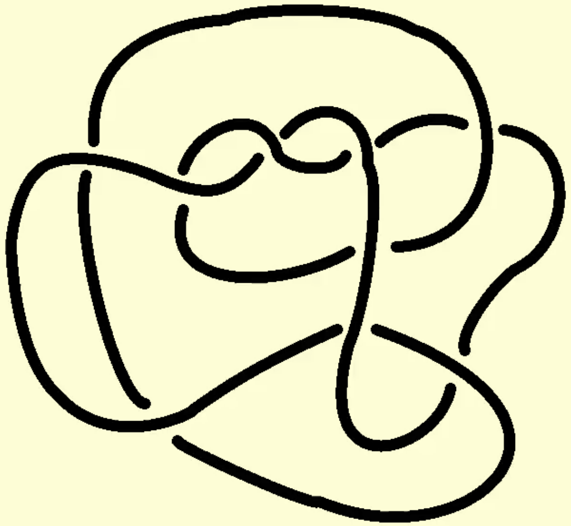
An **invariant** is a function which takes the same value on any equivalent knot.

The *Conway polynomial* is a map

$$\nabla : \text{knots} \rightarrow \text{polynomials}$$



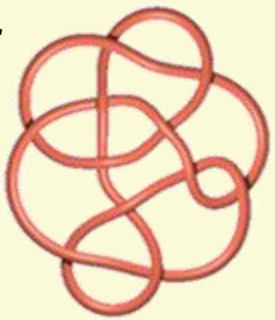
THE REMASTERED DEBUT

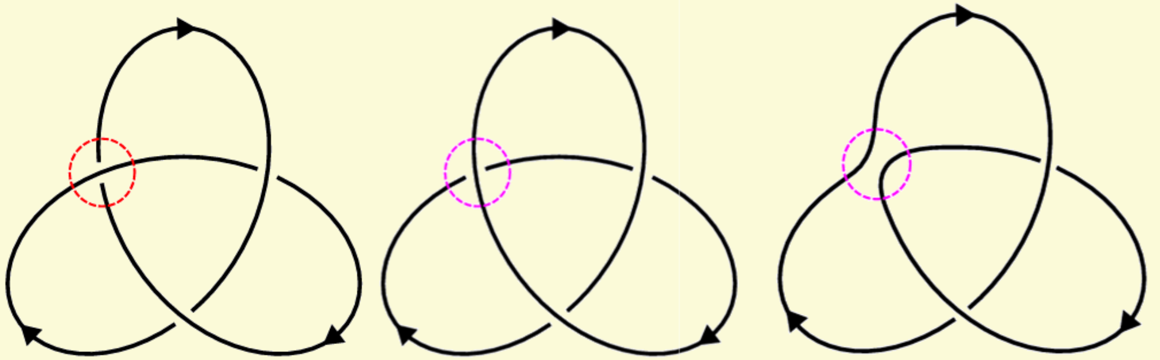


The Alexander-Conway polynomial of the *trefoil* knot is given by

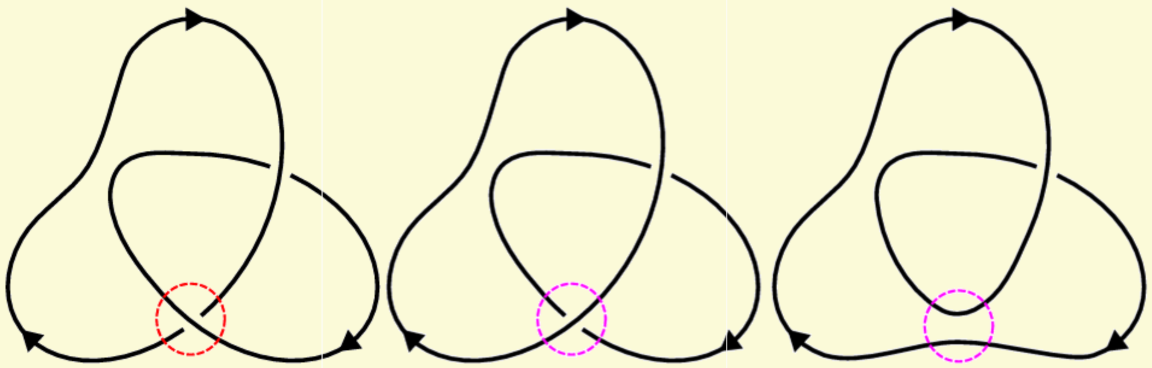
$$\begin{aligned}\nabla_{L_+}(z) &= \nabla_{L_-}(z) + z\nabla_{L_0}(z) \\ &= \nabla_{\bigcirc}(z) + z\nabla_{\widehat{L_+}}(z) \\ &= \nabla_{\bigcirc}(z) + z[\nabla_{\widehat{L_-}}(z) + z\nabla_{\widehat{L_0}}(z)] \\ &= \nabla_{\bigcirc}(z) + z[\nabla_{\bigcirc\bigcirc}(z) + z\nabla_{\bigcirc}(z)] \\ &= 1 + z[0 + z] \\ \nabla(z) &= 1 + z^2.\end{aligned}$$

The *Conway knot* has the same polynomial as the unknot.





from *Alexander-Conway and bracket polynomials of pretzel links P(1,1,n)*



After trying to understand endgames of **Go**, Conway invented the following number system:

- 0 is represented { | }
- If L and R are sets of numbers with $L < R$, then $\alpha = \{L|R\}$ is the 'simplest' number such that $L < \alpha < R$.

THE HIGH CONCEPT ALBUM

$\{\,0\,|\quad\} = 1$ $\qquad$ $\{\quad|\,0\,\} = -1$

$\{\,1\,|\quad\} = 2$ $\qquad$ $\{\quad|-1\} = -2$

$\{\,2\,|\quad\} = 3$ $\qquad$ $\{\quad|-2\} = -3$

$\vdots$ $\qquad$ $\vdots$

$\{\,0\,|\,1\,\} = \frac{1}{2}$ $\qquad$ $\left\{0\,\middle|\,\frac{1}{2}\right\} = \frac{1}{4}$

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THE HIGH CONCEPT ALBUM

$$\{1,2,3,\ldots|\ \} = \omega$$

$$\{\omega|\ \} = \omega + 1$$
$$\{\omega + 1|\ \} = \omega + 2$$
$$\vdots$$
$$\{\omega, \omega + 1, \omega + 2, \ldots|\ \} = 2\omega$$

$$\{\omega, 2\omega, 3\omega, \ldots|\ \} = \omega^2$$

$$\{\omega, \omega^2, \omega^3, \ldots|\ \} = \omega^\omega$$

$$\{\omega, \omega^\omega, \omega^{\omega^\omega}, \ldots|\ \} = \epsilon_0$$

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THE HIGH CONCEPT ALBUM

$$\left\{ 0 \middle| 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots \right\} = \frac{1}{\omega}$$

$$\{1,2,3, \dots | \omega \} = \omega - 1$$

$$\{1,2,3, \dots | \omega, \omega - 1, \dots \} = \frac{\omega}{2}$$

These numbers are a **totally ordered** (proper class) **field.**

We can prove identities like

$$\log \omega = \omega^{1/\omega}$$

SIMPLE GROUPS AND THE MONSTER

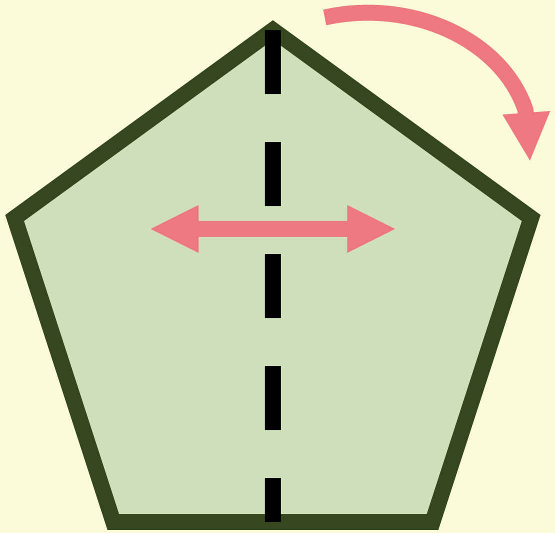
A **group** G describes the symmetries of an object.

- if $g, h \in G$, then $g \circ h \in G$
- $\mathrm{id} \in G$
- if $g \in G$, then $g^{-1} \in G$

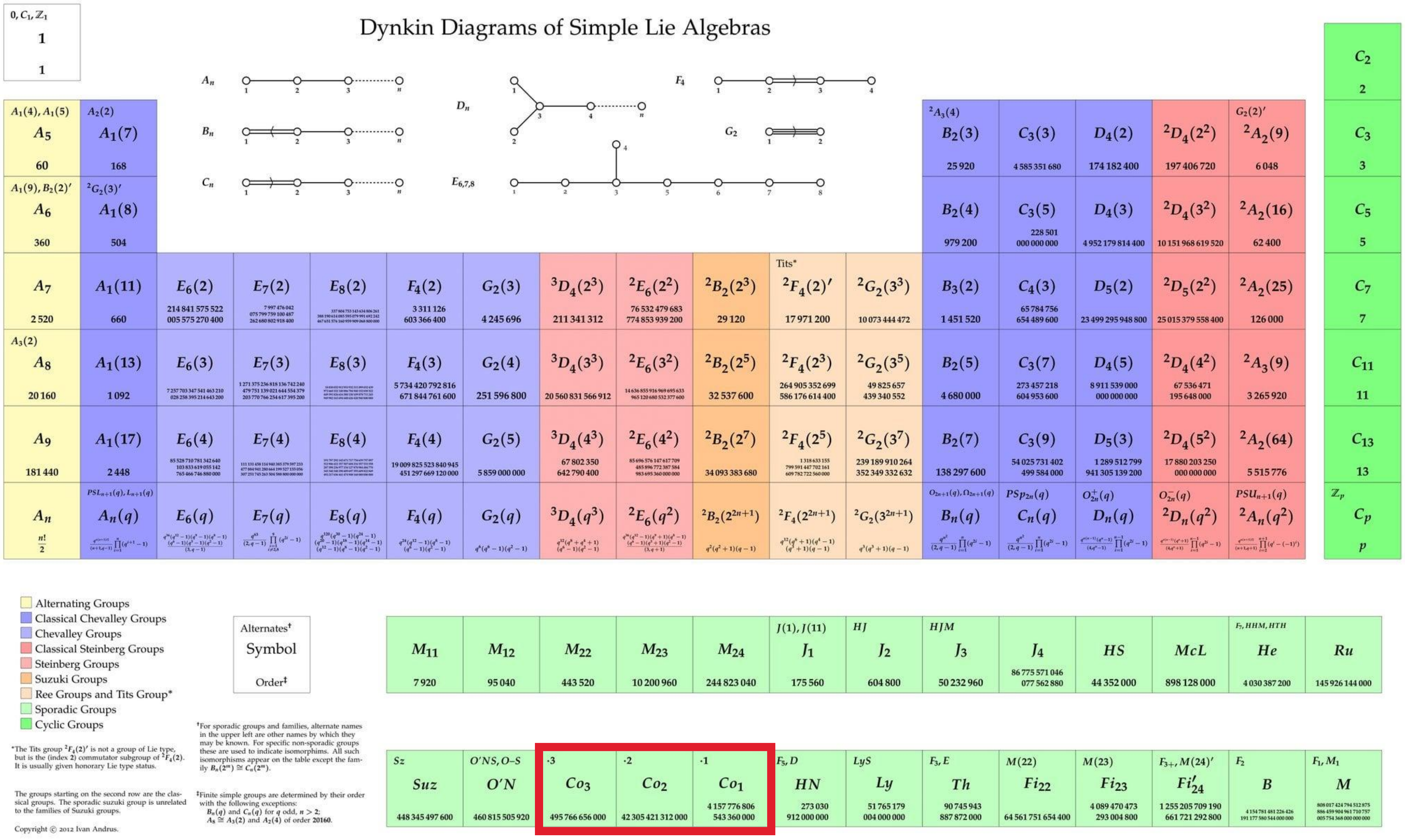
A **normal subgroup** is a subgroup $N \leq G$ such that

$$gNg^{-1} = N$$

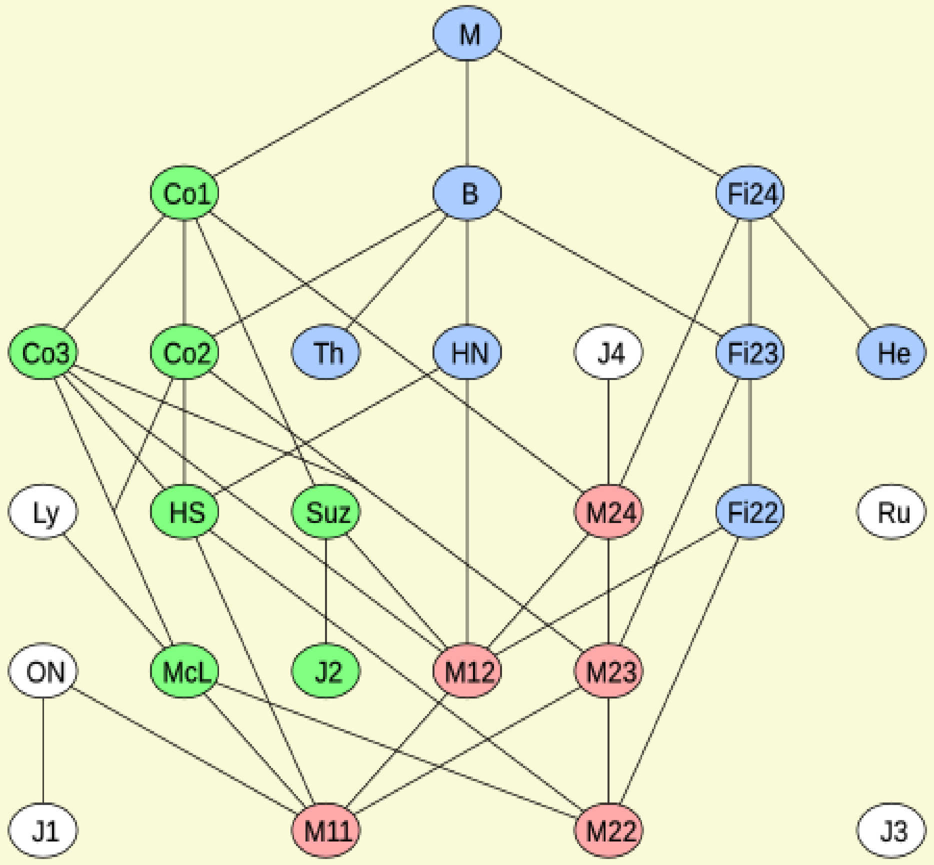
A group is **simple** if it has no normal subgroups.



The Periodic Table Of Finite Simple Groups



$|M| = 808017424794512875886459904961710757005754368000000000$



A **matrix representation** of a group is a map $g \mapsto [g]$, an $n \times n$ matrix over $\mathbb{C}$ with

$$g \circ h = [g][h].$$

For the Monster Group, the smallest rep has $n = 196883$.

In the theory of modular forms, the j-**invariant** is a modular form with Fourier expansion

$$j(\tau) = q^{-1} + 196884q + 21493760q^2 + \cdots$$

Conway did early work on the theory connecting these values, completed by Borcherds ('92).

Other things I didn't mention

- Geometry
Conway polyhedral notation, the grand antiprism, tesellations

- Combinatorial game theory
Sprouts, Philosopher's Football, Domineering strategy

- Number systems
Chained arrow notation, icosians, Waring's conjecture for (5,37)

- Angel vs Devil problem

- Chained arrow notation

- Thrackle conjecture

- Free will theorem

THANKS FOR LISTENING